The Multiscale Entanglement Renormalizaton Ansatz

In ten minutes or less

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G. Alvarez

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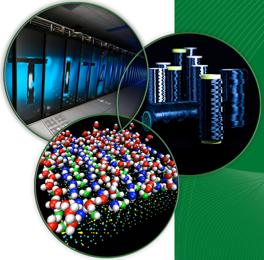

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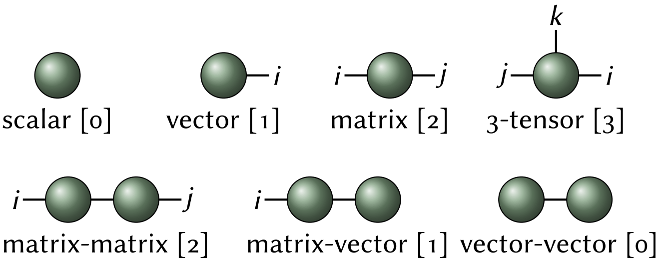
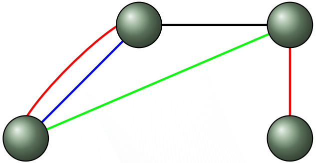
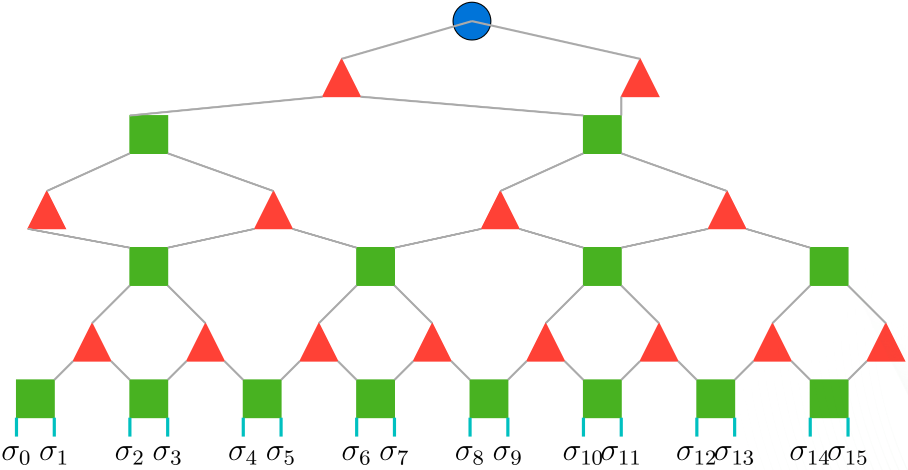
MERA is a Tensor Network

MERA stands for *multiscale entanglement renormalization ansatz* [Vidal, 2008]. MERA is the form of the wavefuction of a quantum problem. This wavefuction belongs to the class of tensor networks.





What is MERA?



A 1D binary MERA $\psi_{\sigma_0,\sigma_1,\sigma_2,\cdots}$ [Evenbly and Vidal, 2009]

What is MERA?

MERA stands for *multiscale entanglement renormalization ansatz* [Vidal, 2008]. MERA is the form of the solution to a quantum problem. This solution form belongs to the class of tensor networks. An algorithm is then applied to obtain the actual values in the MERA and solve the problem, and obtain $\psi_{\sigma_0,\sigma_1,\sigma_2,\cdots}$.

What problems? Finding the ground state of a strongly correlated Hamiltonian, finding a quantum circuit in quantum computing, finding the time evolution of a given quantum state.

Why MERA for Quantum Materials?

Why MERA? Because MERA can systematically and with bounded errors solve many local Hamiltonians in any dimensions, overcoming the limitations of the DMRG.
MERA is not just another variational method. It can be rigorously shown that a polynomial-time truncated MERA tends to the correct solution. And we can estimate the errors made by the truncation.
Yet in 1D, MERA scaling goes like $m,^{28}$ where m is the number of states kept, whereas DMRG goes like $m.^{3}$ In two dimensions, MERA has polynomial scaling and DMRG exponential scaling, but MERA today is slower in practice. For dimensions higher than one, we can improve MERA but not DMRG. To improve MERA, we have a long way to go and tons of work....

Why MERA at ORNL?

There is a large amount of work to be done for MERA to be usable in condensed matter problems. This work aligns well with ORNL's interests and strengths, such as algorithm developement, software development, use of new computer architectures, use of Summit. What's the work that needs to be done?

1. Accelerate tensor-contractions.
2. Implement more geometries: 3D hypercube, triangular.
3. Implement more aries like ternary MERA. Only binary MERA is done.
4. Implement more Models. Only Heisenberg spin 1/2 is done.
5. Make use of local symmetries.
6. Handle fermionic models via "diamond" tensors.

Entropy of Ground State vs. Entropy of Ansatz

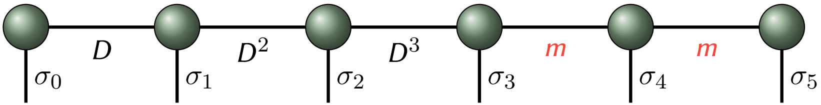
We are going to work with a class $\mathcal{C}$ of strongly correlated Hamiltonians that are short-ranged in some d dimensional geometry and that follow the corrected area law.
The ground state of a Hamiltonian $H \in \mathcal{C}$ in d has[1] the entaglement entropy S_{exact}

dimension	Non-critical	Critical
d	L^{d-1}	$L^{d-1} \ln L$

The ansatz needs to have enough entropy; else the computation will be exponential.

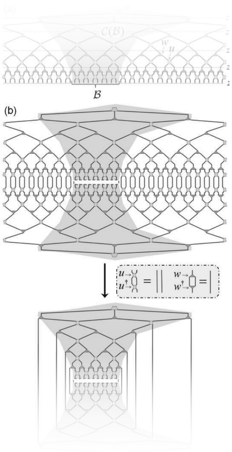
[1]A system is critical if it is gapless *and* $d - d_\Gamma = 1$, where d is the dimension of the geometry, and d_Γ the dimension of the Fermi surface. See citation at the end.

Matrix Product States and DMRG



The entaglement entropy of a MPS of length L^d and bond dimension m is $S_{\text{MPS}}(L^d, m) \propto \ln(m)$. We match $S_{\text{exact}} = L^{d-1} \ln L = S_{\text{MPS}}(L^d, m) = \ln(m)$, to obtain the m required to simulate a problem with MPS or DMRG is as follows:

dimension	Non-critical	Critical
$d = 1$	constant	L
$d > 1$	$\exp(L^{d-1})$	$L^{L^{d-1}}$



MERA

The entaglement entropy of MERA[2] with length L, dimension $d = 1$, and bond dimension m is $S_{\text{MERA}}(L, d = 1, m) \propto \ln(L) \ln(m)$. For dimension $d > 1$ $S_{\text{MERA}}(L, d, m) \propto L^{d-1} \ln(m)$. We match $S_{\text{exact}} = L^{d-1} \ln L = S_{\text{MERA}} = L^{d-1} \ln(m)$ to obtain the m required to simulate a problem with MERA. The answer is shown in the table; see also [Evenbly and Vidal, 2014].

dimension	Non-critical	Critical
$d = 1$	—	constant
$d > 1$	constant	L

scale invariant MERA

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Finding the Ground State MERA

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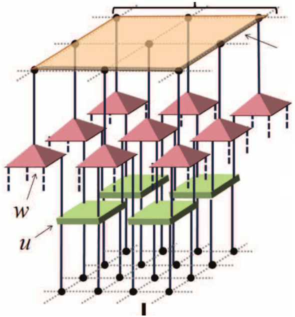
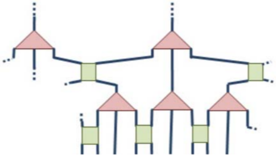
Geometries, Aries, and Models



Dimensions: 2d MERA



Aries: Ternary 1d MERA

Models:
Heisenberg $S = 1/2$, t-J Model, Hubbard, ...

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Summary and Outlook

MERA [Vidal, 2008] is a tensor network and the form of the solution to a quantum problem. It can represent the wavefunction of many quantum Hamiltonians in any dimension.

- This talk will be posted at `https://g1257.github.io/talks/`
- Our MERA++ software is at `https://code.ornl.gov/gonzalo_3/merapp/tree/features` and at `https://github.com/g1257/merapp`
- Our ExaTN software is at `https://code.ornl.gov/qci/exaTN/`

LDRD Collaborators: Eugene Dumitrescu, Dmitry Liakh, Alex McCaskey (PI), and Tiffany Mintz

Credits

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References

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